

Appendix 24A: Hedging with Futures Contracts

Macrohedging with Futures

The number of futures contracts that an FI should buy or sell in a macrohedge depends on the size and direction of its interest rate risk exposure and the return-risk trade-off from fully or selectively hedging that risk. Chapter 23 showed that an FI's net worth exposure to interest rate shocks was directly related to its leverage adjusted duration gap as well as its asset size. Again, this is:

$$\Delta E = -[D_A - kD_L] \times A \times \frac{\Delta R}{1 + R}$$

where

ΔE = Change in an FI's net worth

D_A = Duration of its asset portfolio

D_L = Duration of its liability portfolio

k = Ratio of an FI's liabilities to assets (L/A)

A = Size of an FI's asset portfolio

$\frac{\Delta R}{1 + R}$ = Shock to interest rates

Example 24-3 Calculation of Change in FI Net Worth as Interest Rates Rise

To see how futures might fully hedge a positive or negative portfolio duration gap, consider the following FI where:

$$D_A = 5 \text{ years}$$

$$D_L = 3 \text{ years}$$

Suppose that on May 29, 2002, the FI manager receives information from an economic forecasting unit that interest rates are expected to rise from 10 to 11 percent. That is:

$$\Delta R = 1\% = .01$$

$$1 + R = 1.10$$

The FI's initial balance sheet is:

Assets (in millions)	Liabilities (in millions)
$A = \$100$	$L = \$ 90$
<hr/>	$E = \quad 10$
$\$100$	<hr/>
	$\$100$

Therefore k equals L/A equals $90/100$ equals 0.9 .

The FI manager wants to calculate the potential loss to the FI's net worth (E) if the forecast of rising rates proves to be true. As we showed in Chapter 23:

$$\Delta E = -[D_A - kD_L] \times A \times \frac{\Delta R}{1 + R}$$

so that

$$\Delta E = -[5 - (.9)(3)] \times \$100 \times \frac{.01}{1.1} = -\$2.091 \text{ million}$$

The FI could expect to lose \$2.091 million in net worth if the interest rate forecast turns out to be correct. Since the FI started with a net worth of \$10 million, the loss of \$2.091 million is almost 21 percent of its initial net worth position. Clearly, as this example illustrates, the impact of the rise in interest rates could be quite threatening to the FI and its insolvency risk exposure.

The Risk-Minimizing Futures Position

The FI manager's objective to fully hedge the balance sheet exposure would be fulfilled by constructing a futures position such that if interest rates do rise by 1 percent to 11 percent, as in the prior example, the FI will make a gain on the futures position that just offsets the loss of balance sheet net worth of \$2.091 million.

When interest rates rise, the price of a futures contract falls since its price reflects the value of the underlying bond that is deliverable against the contract. The amount by which a bond price falls when interest rates rise depends on its duration. Thus, we expect the price of the 20-year T-bond futures contract to be more sensitive to interest rate changes than the price of the 3-month T-bill futures contract since the former futures price reflects the price of the 20-year T-bond deliverable on contract maturity. Thus, the sensitivity of the price of a futures contract depends on the duration of the deliverable bond underlying the contract, or:

$$\frac{\Delta F}{F} = -D_F \frac{\Delta R}{1 + R}$$

where

ΔF = Change in dollar value of futures contracts

F = Dollar value of the initial futures contracts

D_F = Duration of the bond to be delivered against the futures contracts such as a 20-year, 8 percent coupon T-bond

ΔR = Expected shock to interest rates

$1 + R$ = 1 plus the current level of interest rates

This can be rewritten as:

$$\Delta F = -D_F \times F \times \frac{\Delta R}{1 + R}$$

The left side of this expression (ΔF) shows the dollar gain or loss on a futures position when interest rates change.

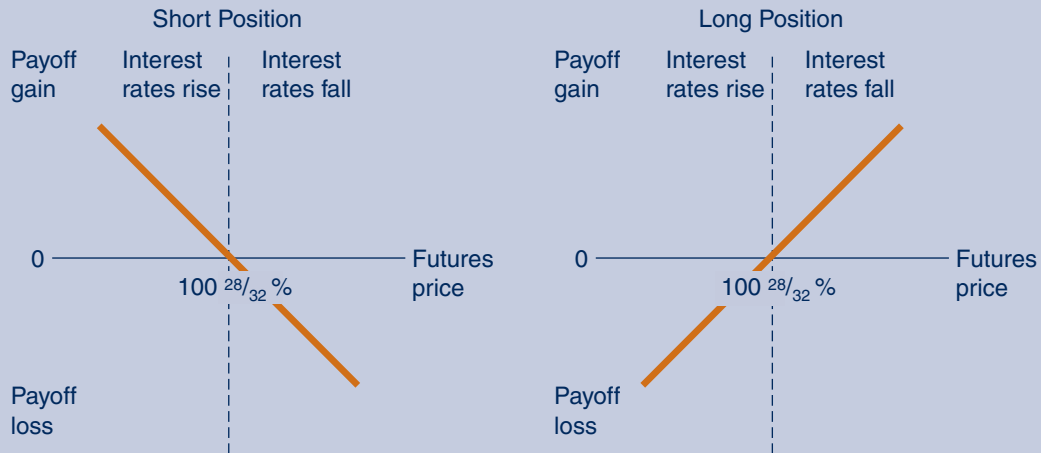
To see this dollar gain or loss more clearly, we can decompose the initial dollar value position in futures contracts, F , into its two component parts:

$$F = N_F \times P_F$$

The dollar value of the outstanding futures position depends on the number of contracts bought or sold (N_F) and the price of each contract (P_F). N_F is positive when the futures contracts are bought and is assigned a negative value when contracts are sold.

Futures contracts are homogeneous in size. Thus, futures exchanges sell T-bond futures in minimum units of \$100,000 of face value; that is, one T-bond futures ($N_F = 1$) equals \$100,000. T-bill futures are sold in larger minimum units: one T-bill future ($N_F = 1$) equals \$1,000,000. The price of each contract quoted in the newspaper is the price per \$100 of face value for delivering the underlying bond. Looking at Figure 24-1, a price quote of $100\frac{3}{32}$ on May 29, 2002, for the T-bond futures contract maturing in September 2002 means that the buyer is required to pay \$100,875 for one contract.²² The subsequent profit or loss from a position in September 2002 T-bond taken on May 29, 2001, is graphically described in Figure 24-A1. A short position in the futures contract will produce a profit when interest rates rise (meaning that the value of the underlying T-bond decreases). Therefore, a short position in the futures market is the appropriate hedge when the FI stands to lose on the balance sheet if interest rates are expected to rise (e.g., the FI has a positive duration gap). A long position in the futures market produces

22. In practice, the futures price changes day to day and gains or losses would be generated for the seller/buyer over the period between when the contract is entered into and when it matures. Note that the FI could sell contracts in T-bonds maturing at later dates. However, while contracts exist for up to two years into the future, longer-term contracts tend to be infrequently traded and therefore relatively illiquid.

Figure 24-A1 Profit or Loss on a Futures Position in Treasury Bonds Taken on May 29, 2002

a profit when interest rates fall (meaning that the value of the underlying T-bond increases).²³ Therefore, a long position is the appropriate hedge when the FI stands to lose on the balance sheet if interest rates are expected to fall (e.g., has a negative duration gap). If, at maturity (in September 2002), the price quote on the T-bond futures contract were $100\frac{28}{32}$ the buyer would pay \$100,875 to the seller and the futures seller would deliver one \$100,000, 20-year, 8 percent T-bond to the futures buyer.

We can now solve for the number of futures contracts to buy or sell to fully macro-hedge an FI's on-balance-sheet interest rate risk exposure. We have shown that:

1. *Loss on balance sheet.* The loss of net worth for an FI when rates change is equal to:

$$\Delta E = -(D_A - kD_L)A \frac{\Delta R}{1 + R}$$

2. *Gain off balance sheet on futures.* The gain off balance sheet from selling futures is equal to:²⁴

$$\Delta F = -D_F (N_F \times P_F) \frac{\Delta R}{1 + R}$$

Fully hedging can be defined as buying or selling a sufficient number of futures contracts (N_F) so that the loss of net worth on the balance sheet (ΔE) when interest rates change is just offset by the gain from off-balance-sheet buying or selling of futures, (ΔF), or:

$$\Delta F = \Delta E$$

Substituting in the appropriate expressions for each:

$$-D_F (N_F \times P_F) \frac{\Delta R}{1 + R} = -(D_A - kD_L)A \frac{\Delta R}{1 + R}$$

canceling $\Delta R/(1 + R)$ on both sides:²⁵

$$D_F (N_F \times P_F) = (D_A - kD_L)A$$

23. Notice that if rates move in an opposite direction from that expected, losses are incurred on the futures position. That is, if rates rise and futures prices drop, the long hedger loses. Similarly, if rates fall and futures prices rise, the short hedger loses. However, such losses are offset by gains on their cash market positions. Thus, the hedger is still protected.

24. When futures prices fall, the buyer of the contract compensates the seller, here the FI. Thus, the FI gains when the prices of futures fall.

25. This amounts to assuming that the interest changes of the cash asset position match those of the futures position; that is, there is no basis risk. This assumption is relaxed later.

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Solving for N_F (the number of futures to buy or sell) gives:

$$N_F = \frac{(D_A - kD_L)A}{D_F \times P_F}$$

Short Hedge

An FI takes a short position in a futures contract when rates are expected to rise; that is, the FI loses net worth on its balance sheet if rates rise, so it seeks to hedge the value of its net worth by selling an appropriate number of futures contracts.

Example 24-4 Macrohedge of Interest Rate Risk Using a Short Hedge

From the equation for N_F , we can now solve for the correct number of futures contracts to sell (N_F) in the context of Example 24-3 where the FI was exposed to a balance sheet loss of net worth (ΔE) amounting to \$2.091 million when interest rates rose. In that example:

$$\begin{aligned} D_A &= 5 \text{ years} \\ D_L &= 3 \text{ years} \\ k &= .9 \\ A &= \$100 \text{ million} \end{aligned}$$

Suppose the current futures price quote is \$97 per \$100 of face value for the benchmark 20-year, 8 percent coupon bond underlying the nearby futures contract, the minimum contract size is \$100,000, and the duration of the deliverable bond is 9.5 years. That is:

$$\begin{aligned} D_F &= 9.5 \text{ years} \\ P_F &= \$97,000 \end{aligned}$$

Inserting these numbers into the expression for N_F , we can now solve for the number of futures to sell:

$$\begin{aligned} N_F &= \frac{(5 - (.9)(3)) \times \$100 \text{ million}}{9.5 \times \$97,000} \\ &= \frac{\$230,000,000}{\$921,500} \\ &= 249.59 \text{ contracts to be sold} \end{aligned}$$

Since the FI cannot sell a part of a contract, the number of contracts should be rounded down to the nearest whole number, or 249 contracts.²⁶

Next, we verify that selling 249 T-bond futures contracts will indeed hedge the FI against a sudden increase in interest rates from 10 to 11 percent, or a 1 percent interest rate shock.

On Balance Sheet

As shown above, when interest rates rise by 1 percent, the FI loses \$2.091 million in net worth (ΔE) on the balance sheet:

$$\begin{aligned} \Delta E &= -(D_A - kD_L)A \frac{\Delta R}{1 + R} \\ -\$2.091 \text{ million} &= -(5 - (.9)(3)) \times \$100 \text{ million} \times \left(\frac{.01}{1.1} \right) \end{aligned}$$

26. The reason for rounding down rather than rounding up is technical. The target number of contracts to sell is that which minimizes interest rate risk exposure. By slightly underhedging rather than overhedging, the FI can generate the same risk exposure level but the underhedging policy produces a slightly higher return.

Table 24–A1 On- and Off-Balance-Sheet Effects of a Macrohedge Hedge

	On Balance Sheet	Off Balance Sheet
Begin hedge $t = 0$	Equity value of \$10 million exposed to impact of rise in interest rates.	Sell 249.59 T-bond futures contracts at \$97,000. Underlying T-bond coupon rate is 8%.
End hedge $t = 1$ day	Interest rates rise on assets and liabilities by 1%. Opportunity loss on-balance-sheet: $\Delta E = -[5 - .9(3)] \times \$100\text{m} \times \frac{.01}{1.1}$ $= -\$2.091 \text{ million}$	Buy 249.59 T-bond futures (closes out futures position). Real gain on futures hedge: $\Delta F = -9.5 \times (-249.59 \times \$97,000) \times \frac{.01^*}{1.1}$ $= \$2.091 \text{ million}$

*Assuming no basis risk and no contract “rounding.”

Off Balance Sheet

When interest rates rise by 1 percent, the change in the value of the futures position is:

$$\begin{aligned}\Delta F &= -D_F(N_F \times P_F) \frac{\Delta R}{1 + R} \\ &= -9.5(-249 \times \$97,000) \left(\frac{.01}{1.1} \right) \\ &= \$2.086 \text{ million}\end{aligned}$$

The value of the off-balance-sheet futures position (ΔF) falls by \$2.086 million when the FI sells 249 futures contracts in the T-bond futures market. Such a fall in value of the futures contracts means a positive cash flow to the futures seller as the buyer compensates the seller for a lower futures price through the marking-to-market process. This requires a cash flow from the buyer’s margin account to the seller’s margin account as the price of a futures contract falls. Thus, as the seller of the futures, the FI makes a gain of \$2.086 million. As a result, the net gain/loss on and off the balance sheet is:

$$\Delta E + \Delta F = -\$2.091 \text{ m} + \$2.086 \text{ m} = -\$0.005 \text{ million}$$

This small remaining net loss of \$.005 million to equity or net worth reflects the fact that the FI could not achieve the perfect hedge—even in the absence of basis risk—as it needed to round down the number of futures to the nearest whole contract from 249.59 to 249 contracts. Table 24–A1 summarizes the key features of the hedge (assuming no rounding of futures contracts).

The Problem of Basis Risk

Because spot bonds and futures on bonds are traded in different markets, the shift in yields, $\Delta R/(1 + R)$, affecting the values of the on-balance-sheet cash portfolio may differ from the shifts in yields, $\Delta R_F/(1 + R_F)$, affecting the value of the underlying bond in the futures contract; that is, changes in spot and futures prices or values are not perfectly correlated. This lack of perfect correlation is called *basis risk*. In the previous section, we assumed a simple world of no basis risk in which $\Delta R/(1 + R) = \Delta R_F/(1 + R_F)$.

Basis risk occurs for two reasons. First, the balance sheet asset or liability being hedged is not the same as the underlying security on the futures contract. For instance, in Example 24–4 we hedged interest rate changes on the FI’s entire balance sheet with T-bond futures contracts written on 20-year maturity bonds with a duration of 9.5 years. The interest rates on the various assets and liabilities on the FI’s balance sheet

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and the interest rates on 20-year T-bonds do not move in a perfectly correlated (or one-to-one) manner. The second source of basis risk comes from the difference in movements in spot rates versus futures rates. Because spot securities (e.g., government bonds) and futures contracts (e.g., on the same bonds) are traded in different markets, the shift in spot rates may differ from the shift in futures rates (i.e., they are not perfectly correlated).

To solve for the risk minimizing number of futures contracts to buy or sell, N_F , while accounting for greater or less rate volatility and hence price volatility in the futures market relative to the spot or cash market, we look again at the FI's on-balance-sheet interest rate exposure:

$$\Delta E = -(D_A - kD_L) \times A \times \Delta R / (1 + R)$$

and its off-balance-sheet futures position:

$$\Delta F = -D_F(N_F \times P_F) \times \Delta R_F / (1 + R_F)$$

Setting:

$$\Delta E = \Delta F$$

and solving for N_F , we have:

$$N_F = \frac{(D_A - kD_L) \times A \times \Delta R / (1 + R)}{D_F \times P_F \times \Delta R_F / (1 + R_F)}$$

Let br reflect the relative sensitivity of rates underlying the bond in the futures market relative to interest rates on assets and liabilities in the spot market, i.e., $br = (\Delta R_F / (1 + R_F)) / (\Delta R / (1 + R))$. Then the number of futures contracts to buy or sell is:

$$N_F = \frac{(D_A - kD_L)A}{D_F \times P_F \times br}$$

The only difference between this and the previous formula is an adjustment for basis risk (br), which measures the degree to which the futures price (yields) moves more or less than spot bond price (yields).

Microhedging with Futures

The number of futures contracts that an FI should buy or sell in a microhedge depends on the interest rate risk exposure created by a particular asset or liability on the balance sheet. The key is to take a position in the futures market to offset a loss on the balance sheet due to a move in interest rates with a gain in the futures market. Recall from Chapter 23 that the change in value of an asset or liability on the FI's balance sheet from a change in interest rates equals:

$$\Delta P = -D \times P \times \frac{\Delta R}{1 + R}$$

We can now solve for the number of futures contracts to buy or sell to microhedge an FI's assets or liabilities. We have shown the following:

1. *Loss on the balance sheet* from a change in interest rates is:

$$\Delta P = -D \times P \times \frac{\Delta R}{1 + R}$$

2. *Gain off the balance sheet* from a position in the futures contract is:

$$\Delta F = -D_F \times (N_F \times P_F) \times \frac{\Delta R_F}{1 + R_F}$$

Hedging can be defined as buying or selling a sufficient number of futures contracts (N_F) so that the loss on the balance sheet (ΔP) due to rate changes is just offset by a gain off the balance sheet on the position in futures contracts (ΔF), or:

$$\Delta F = \Delta P$$

Substituting the appropriate expressions for each:

$$-D_F \times (N_F \times P_F) \times \frac{\Delta R_F}{1 + R_F} = -D \times P \times \frac{\Delta R}{1 + R}$$

Remembering that basis risk, $br = (\Delta R_F / (1 + R_F)) / (\Delta R / (1 + R))$, is the measure of relative sensitivity of rates underlying the bond in the futures market relative to interest rates on assets and liabilities in the spot market:

$$-D_F \times N_F \times P_F \times br = -D \times P$$

Solving for N_F (the number of futures contracts to buy or sell):

$$N_F = \frac{D \times P}{D_F \times P_F \times br}$$

Appendix 24B: Hedging with Options

Macrohedging with Options

Chapter 23 showed that an FI's net worth exposure to an interest rate shock could be represented as:

$$\Delta E = -(D_A - kD_L) \times A \times \frac{\Delta R}{1 + R}$$

where

$$\begin{aligned} \Delta E &= \text{Change in the FI's net worth} \\ (D_A - kD_L) &= \text{FI's duration gap} \\ A &= \text{Size of the FI's assets} \\ \frac{\Delta R}{1 + R} &= \text{Size of the interest rate shock} \\ k &= \text{FI's leverage ratio } (L/A) \end{aligned}$$

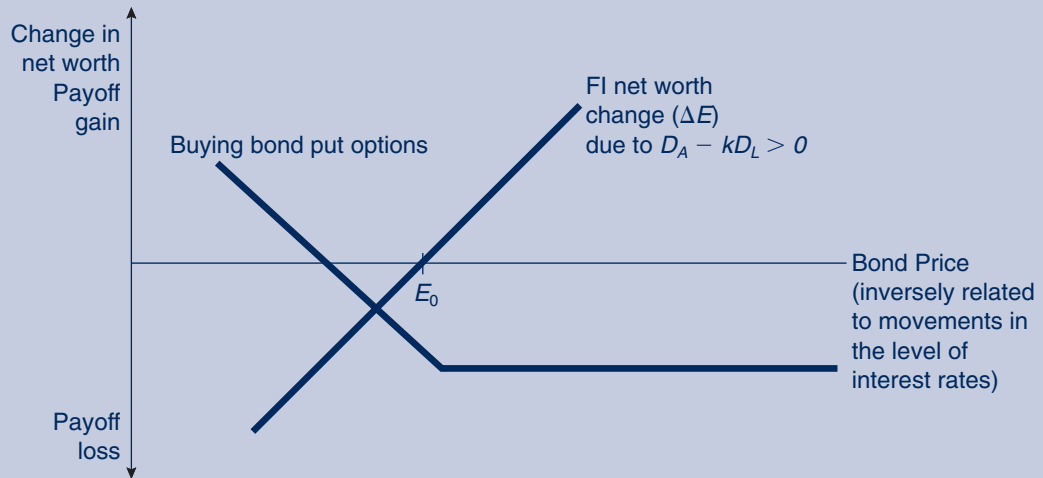
Suppose the FI manager wishes to determine the optimal number of put options to buy to insulate the FI against rising rates. An FI with a positive duration gap (see Figure 24–B1) would lose on-balance-sheet net worth when interest rates rise. In this case, the FI manager would buy put options.²⁷ That is, the FI manager wants to adopt a put option position to generate profits that just offset the loss in net worth due to an interest rate shock (where E_0 is the FI's initial equity (net worth) position in Figure 24–B1).

Let ΔP be the total change in the value of the put option position in T-bonds. This can be decomposed into:

$$(1) \quad \Delta P = (N_p \times \Delta p)$$

where N_p is the number of \$100,000 put options on T-bond contracts to be purchased (the number for which we are solving) and Δp is the change in the dollar value for each \$100,000 face value T-bond put option contract.

27. Conversely, an FI with a negative duration gap would lose on-balance-sheet net worth when interest rates fall. In this case, the FI manager wants to buy call options to generate profits to offset the loss in net worth due to an interest rate shock.

Figure 24-B1 Buying Put Options to Hedge the Interest Rate Risk Exposure of the FI

The change in the dollar value of each contract (Δp) can be further decomposed into:

$$(2) \quad \Delta p = \frac{dp}{dB} \times \frac{dB}{dR_b} \times \Delta R_b$$

This decomposition needs some explanation. The first term (dp/dB) shows the change in the value of a put option for each \$1 dollar change in the underlying bond. This is called the *delta of an option* (δ) and lies between 0 and 1. For put options, the delta has a negative sign since the value of the put option falls when bond prices rise.²⁸ The second term (dB/dR_b) shows how the market value of a bond changes if interest rates rise by one basis point. This value of one basis point term can be linked to duration. Specifically, we know from Chapter 3 that:

$$(3) \quad \frac{dB}{B} = -MD \times dR_b$$

That is, the percentage change in the bond's price for a small change in interest rates is proportional to the bond's modified duration (MD). Equation (3) can be rearranged by cross multiplying as:

$$(4) \quad \frac{dB}{dR_b} = -MD \times B$$

Thus, the term dB/dR_b is equal to minus the modified duration on the bond (MD) times the current market value of the T-bond (B) underlying the put option contract. As a result, we can rewrite equation (2) as:

$$(5) \quad \Delta p = [(-\delta) \times (-MD) \times B \times \Delta R_b]$$

where ΔR_b is the shock to interest rates (i.e., the number of basis points by which bond rates change). Since from Chapter 3 we know that $MD = D/(1 + R_b)$, we can rewrite equation (5) as:

$$(6) \quad \Delta p = \left[(-\delta) \times (-D) \times B \times \frac{\Delta R_b}{1 + R_b} \right]$$

28. For call options, the delta has a positive sign since the value of the call rises when bond prices rise. As we proceed with the derivation, we examine only the case of a hedge using a put option contract (i.e., the FI has a positive duration gap and expects interest rates to rise). For a hedge with a call option contract (i.e., the FI has a negative duration gap), the derivation below changes only in that the sign on the delta is reversed (from negative to positive).

Thus, the change in the total value of a put position²⁹ (ΔP) is

$$(7) \quad \Delta P = N_p \times \left[\delta \times D \times B \times \frac{\Delta R_b}{1 + R_b} \right]$$

The term in brackets is the change in the value of one \$100,000 face-value T-bond put option as rates change, and N_p is the number of put option contracts.

To hedge net worth exposure, we require the profit on the off-balance-sheet put options (ΔP) to just offset the loss of on-balance-sheet net worth ($-\Delta E$) when interest rates rise (and thus, bond prices fall). That is:³⁰

$$\Delta P = -\Delta E$$

$$N_p \times \left[\delta \times D \times B \times \frac{\Delta R_b}{1 + R_b} \right] = [D_A - kD_L] \times A \times \frac{\Delta R}{1 + R}$$

Substituting br for $(\Delta R_b/(1 + R_b))/(\Delta R/(1 + R))$, we get:

$$N_p \times [\delta \times D \times B \times br] = [D_A - kD_L] \times A$$

Solving for N_p —the number of put options to buy—we have:

$$(8) \quad N_p = \frac{[D_A - kD_L] \times A}{[\delta \times D \times B \times br]}$$

Example 24-5 Macrohedge of Interest Rate Risk Using a Put Option

Suppose, as in Example 24-4, an FI's balance sheet is such that $D_A = 5$, $D_L = 3$, $k = .9$, and $A = \$100$ million. Rates are expected to rise from 10 to 11 percent over the next six months, which would result in a \$2.09 million loss in net worth to the FI. Suppose also that δ of the put option is .5, which indicates that the option is close to being in the money, $D = 8.82$ for the bond underlying the put option contract, the current market value of \$100,000 face value of long-term Treasury bonds underlying the option contract, B , equals \$97,000, the rate of return on the bond, R_b , is 10 percent, and basis risk, br , is 0.92. Solving for N_p , the number of put option contracts to buy:

$$N_p = \frac{\$230,000,000}{[.5 \times 8.82 \times \$97,000 \times 0.92]} = \frac{\$230,000,000}{\$393,548.4}$$

$$= 584.4262 \text{ contracts}$$

If the FI slightly underhedges, this will be rounded down to 584 contracts. If on-balance-sheet rates increase from 10 to 11 percent on the bond underlying the put option and interest rates (R) increase from 10 to 10.92 percent, i.e., $br = 0.92$, the value of the FI's put options will change by:

$$\Delta P = 584 \times \left[.5 \times 8.82 \times \$97,000 \times \frac{.0092}{1.1} \right] = \$2.09 \text{ million}$$

just offsetting the loss in net worth on the balance sheet.

29. Note that since both the delta and D of the put option and bond have negative signs, their product will be positive. Thus, these negative signs are not shown in the equation to calculate N_p .

30. Note that: $\Delta E = -(D_A - kD_L) \times A \times \frac{\Delta R}{1 + R}$

Thus:

$$-\Delta E = +(D_A - kD_L) \times A \times \frac{\Delta R}{1 + R}$$

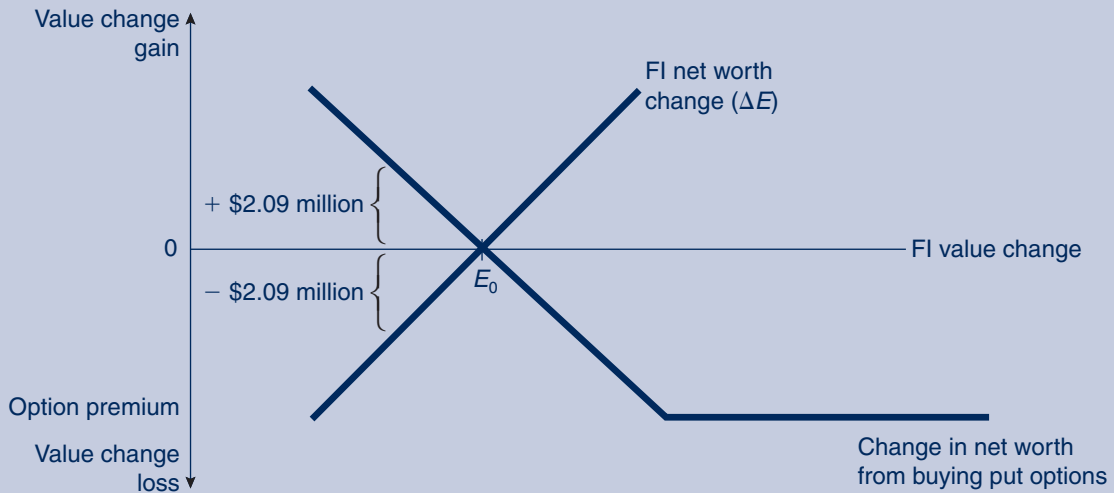
Figure 24–B2 Buying Put Options to Hedge an FI's Interest Rate GAP Risk Exposure

Figure 24–B2 summarizes the change in the FI's overall value from a one percent increase in interest rates and the offsetting change in value from the hedge in the put option market. If rates increase as predicted, the FI's gap exposure results in a decrease in net worth of \$2.09 million. This decrease is offset with a \$2.09 million gain on the put options position held by the FI. Should rates decrease, however, the resulting increase in net worth is not offset by a decrease in an out-of-the-money put option.

Microhedging with Options

Recall from Chapter 3 that for an asset on the FI's balance sheet:

$$(9) \quad \Delta P = -D_A \times P \times \frac{\Delta R}{1 + R}$$

An asset held in an FI's portfolio will lose value if interest rates increase. If the FI has no liability to offset this loss in asset value, the FI's on-balance-sheet net worth will fall (i.e., $\Delta E = \Delta P$). The FI can hedge this interest rate risk, however, by buying a put option off the balance sheet.

As shown earlier, the change in the total value of a put option position (ΔP) is:

$$(10) \quad \Delta P = N_p \times \left[\delta \times D \times B \times \frac{\Delta R_b}{1 + R_b} \right]$$

where B is the value of the bond underlying the option contract, δ is the value change of an option for a \$1 change in the value of the underlying bond, and D is the underlying bond's duration.

To hedge net worth exposure, we require the profit on the off-balance-sheet options to just offset the loss of on-balance-sheet assets when rates change. That is:

$$(11) \quad \Delta P = -\Delta E$$

or:

$$(12) \quad N_p \times \left[\delta \times D \times B \times \frac{\Delta R_b}{1 + R_b} \right] = D_A \times P \times \frac{\Delta R}{1 + R}$$

when hedging interest rate risk on an asset using a put option. Solving for N_p , the number of put options to buy³¹:

$$(13) \quad N_p = \frac{D_A \times P}{\delta \times D \times B \times br}$$

where $br = (\Delta R_b / (1 + R_b)) / (\Delta R / (1 + R))$.

31. For hedging a liability with a call option, the formula is: $N_c = \frac{D_L \times P}{\delta \times D \times B \times br}$